Topic: Implicit differentiation, equation of the tangent line

Question: Find the equation of the tangent line to the curve at the given point.

$$\frac{x^2}{9}+\frac{y^2}{4}=2$$

at $(3,2)$

Answer choices:

A $y=-\frac{2}{3}x+4$

B $y=\frac{2}{3}x-4$

C $y=\frac{2}{3}x+4$

D $y=-\frac{2}{3}x-4$

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Solution: A

To find the tangent line to a curve at a given point, we first need to find the slope, m. The slope is the derivative of the function at the given point, which means that once we know the derivative, we can plug in the given point to find the slope.

Then to find the equation of the tangent line, we can plug the slope m and the given point (x_1, y_1) into the point-slope formula for the equation of a line $(y - y_1) = m(x - x_1)$.

Using implicit differentiation to find the derivative of our function, we get

$$\frac{2x}{9} + \frac{2y}{4}y' = 0$$

Solving for y', we get

$$\frac{2y}{4}y' = -\frac{2x}{9}$$

$$y' = -\frac{2x(4)}{9(2y)}$$

$$y' = -\frac{8x}{18y}$$

$$y' = -\frac{4x}{9y}$$

To find the slope m, we'll plug the given point $(3,2)$ into the derivative.

$$m = -\frac{4(3)}{9(2)}$$

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$$m = -\frac{12}{18}$$

$$m = -\frac{2}{3}$$

Now we'll take the slope $m = -2/3$, the given point $(3,2)$ and the point-slope formula for the equation of a line $(y - y_1) = m(x - x_1)$, and solve for the equation of the tangent line.

$$(y - 2) = -\frac{2}{3}(x - 3)$$

$$y - 2 = -\frac{2}{3}x + 2$$

$$y = -\frac{2}{3}x + 4$$

The equation of the tangent line to the curve

$$\frac{x^2}{9} + \frac{y^2}{4} = 2$$

at the point $(3,2)$ is

$$y = -\frac{2}{3}x + 4$$

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Topic: Implicit differentiation, equation of the tangent line

Question: Find the equation of the tangent line to the curve at the given point.

$$5x^2 + y^2 + 4xy = 1$$

at $(0,1)$

Answer choices:

A $y = 2x - 1$

B $y = -2x + 1$

C $y = -2x - 1$

D $y = 2x + 1$

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Solution: B

To find the tangent line to a curve at a given point, we first need to find the slope, m. The slope is the derivative of the function at the given point, which means that once we know the derivative, we can plug in the given point to find the slope.

Then to find the equation of the tangent line, we can plug the slope m and the given point (x_1, y_1) into the point-slope formula for the equation of a line $(y - y_1) = m(x - x_1)$.

Using implicit differentiation to find the derivative of our function, we get

$$10x + 2yy' + (4)(y) + (4x)(1y') = 0$$

$$10x + 2yy' + 4y + 4xy' = 0$$

Solving for y', we get

$$2yy' + 4xy' = -10x - 4y$$

$$y'(2y + 4x) = -10x - 4y$$

$$y' = -\frac{10x + 4y}{2y + 4x}$$

To find the slope m, we'll plug the given point $(0,1)$ into the derivative.

$$m = -\frac{10(0) + 4(1)}{2(1) + 4(0)}$$

$$m = -\frac{4}{2}$$

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$$m = -2$$

Now we'll take the slope $m = -2$, the given point $(0,1)$ and the point-slope formula for the equation of a line $(y - y_1) = m(x - x_1)$, and solve for the equation of the tangent line.

$$(y - 1) = -2(x - 0)$$

$$y - 1 = -2x$$

$$y = -2x + 1$$

The equation of the tangent line to the curve

$$5x^2 + y^2 + 4xy = 1$$

at the point $(0,1)$ is

$$y = -2x + 1$$

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Topic: Implicit differentiation, equation of the tangent line

Question: Find the equation of the tangent line to the curve at the given point.

$$8 + x^2y^2 - 5xy = 4$$

at $(1,4)$

Answer choices:

A $y = 4x + 8$

B $y = -4x - 8$

C $y = 4x - 8$

D $y = -4x + 8$

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Solution: D

To find the tangent line to a curve at a given point, we first need to find the slope, m. The slope is the derivative of the function at the given point, which means that once we know the derivative, we can plug in the given point to find the slope.

Then to find the equation of the tangent line, we can plug the slope m and the given point (x_1, y_1) into the point-slope formula for the equation of a line $(y - y_1) = m(x - x_1)$.

Using implicit differentiation to find the derivative of our function, we get

$$0 + (2x)\left(y^2\right) + \left(x^2\right)(2yy') - \left[(5)(y) + (5x)(1y')\right] = 0$$

$$2xy^2 + 2x^2yy' - \left(5y + 5xy'\right) = 0$$

$$2xy^2 + 2x^2yy' - 5y - 5xy' = 0$$

Solving for y', we get

$$2x^2yy' - 5xy' = 5y - 2xy^2$$

$$y'\left(2x^2y - 5x\right) = 5y - 2xy^2$$

$$y' = \frac{5y - 2xy^2}{2x^2y - 5x}$$

To find the slope m, we'll plug the given point $(1,4)$ into the derivative.

$$m = \frac{5(4) - 2(1)(4)^2}{2(1)^2(4) - 5(1)}$$

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$$m = \frac{20 - 32}{8 - 5}$$

$$m = \frac{-12}{3}$$

$$m = -4$$

Now we'll take the slope $m = -4$, the given point $(1,4)$ and the point-slope formula for the equation of a line $(y - y_1) = m(x - x_1)$, and solve for the equation of the tangent line.

$$(y - 4) = -4(x - 1)$$

$$y - 4 = -4x + 4$$

$$y = -4x + 8$$

The equation of the tangent line to the curve

$$8 + x^2y^2 - 5xy = 4$$

at the point $(1,4)$ is

$$y = -4x + 8$$

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